Circulation in Bubble Columns: Corrections for Distorted Bubble Shape

N. W. Geary and R. G. Rice
Dept. of Chemical Engineering, Louisiana State University, Baton Rouge, LA 70803

In a recent exchange of correspondence, Hills (1991) provided unpublished data based on resistance probe measurements related to estimating bubble size within swarms of bubbles. In reply to this letter, Rice and Geary (1991) pointed to a new theory (Geary and Rice, 1991) to predict formation size, which they showed could closely match the data of Hills, if the ellipsoidal shape of rising bubbles was taken into account. This explained, in part, why the original circulation theory (Rice and Geary, 1990) could so accurately match the local liquid velocity profiles of Hills (1974), where bubble size predictions were consistently oversized. Thus, two important geometric effects are seen to nearly cancel each other: the actual bubble size is indeed smaller, but the correct turbulence eddy scale should be the larger, major axis of the ellipsoidal-shaped bubble.

In this work, we formalize these corrections, based in part on the recommendations given in the new book by Fan and Tsuchiya (1990), who provided correlations for the aspect ratio based only on a single dimensionless group (the Tadaki number).

In our original (Rice and Geary, 1990) Prandtl model for two-phase turbulence, the locally varying eddy size $\ell(\xi)$ was taken to be proportional to the spherical bubble formation size. This simplification ignores several important characteristics, the most important being that the eddy size will be proportional to the deformed bubble's breadth. This is because the vortices, formed in the wake of a rising bubble, result from boundary-layer separation. If, for instance, the bubble takes an ellipsoidal shape, then the eddy size will be greater than the spherical formation size. For larger bubbles, the pressure differential across a rising bubble tends to deform and squash the bubble. This enlarges the projected area, thus increasing the drag. The overall effect is to limit the rise velocity, so it becomes independent of size. In addition, any increase in vortex size (as it dissipates) was not considered. This phenomenon is also documented by Fan and Tsuchiya (1990), but will not be applied here.

The aspect ratio for a rising, ellipsoidal bubble is defined as the ratio of its height (minor axis) to its breadth (major axis): h/b. Fan and Tsuchiya (1990) reviewed the literature on this subject and the following correlation is recommended:

$$\frac{h}{b}=\begin{cases}1, & Ta<1\\ [0.81+0.206\tanh(2[0.8-\log_{10}Ta])]^3, & 1\le Ta\le 39.8\\ 0.24, & Ta>39.8\end{cases}\tag{1}$$

This relationship is presented in Figure 1, where it is compared to literature data. The Tadaki number is defined as:

$$Ta=g^{1/4}(\rho_L/\sigma)^{3/4}d\cdot U\tag{2}$$

The spherical formation size (d) can be found from the recent theory of Geary and Rice (1991), which compared favorably with a variety of measurements. For large rising bubbles or droplets ($d>3$ mm) deformation is so severe that U becomes independent of size. Levich (1962) derived the following relationship for deformed droplets (or bubbles) which is size-independent:

$$U\approx\left(\frac{4\sigma g}{C_D\cdot\rho_L}\right)^{1/4}\quad Re>850\tag{3}$$

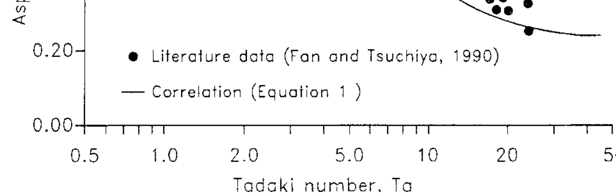

Figure 1. Aspect ratio (*h/b*) as a function of Tadaki number.
From Fan and Tsuchiya (1990).

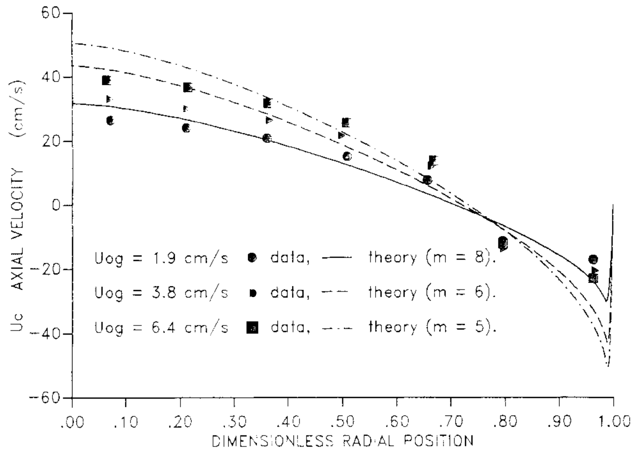

Figure 2. Predicted velocity profiles: Rice-Geary Model (1990) with $\ell \propto b$ vs. Hills data (1974).

This predicts $U = 25.7$ cm/s when $C_D = 0.65$ (Perry, 1974). This value is applicable for most air-water systems. Thus, the formation size and rise velocity can be used to calculate the Tadaki number, from which the major length scale b can be determined (via Eq. 1). In terms of an equivalent sphere diameter d, we have simply:

$$b/d = (h/b)^{-1/3} = f(Ta)^{-1/3} \tag{4}$$

since Eq. 1 shows $h/b = f(Ta)$. Thus, when $Ta > 40$, $b \sim 1.6\ d$.

We now return to the original circulation theory of Rice and Geary (1990) and modify the local turbulence length scale by replacing d with b:

$$\ell(\xi) = b \cdot \epsilon(\xi)/\bar{\epsilon} \tag{5}$$

where $\epsilon(\xi)$ is the locally varying gas voidage, and $\bar{\epsilon}$ is the average voidage. In terms of Tadaki number, this becomes:

$$\ell(\xi) = d \cdot f(Ta)^{-1/3} \cdot \epsilon(\xi)/\bar{\epsilon} \tag{6}$$

Thus, the elementary correction is simply $(h/b)^{-1/3}$, which for $Ta > 40$ implies multiplication by the factor 1.6.

We have incorporated this correction and recomputed the velocity profiles using our previous circulation theory (Rice and Geary, 1990) for the conditions corresponding to the experiments of Hills (1974), as shown in Figure 2. (See Table 1 for parameters and conditions.) The comparison is again quite good. Apparently, the earlier agreement arose from using an oversize bubble diameter predictor. When this size is downscaled (based on the new formation theory by Geary and Rice, 1991) and then corrections applied for the ellipsoidal shape, a good comparison with Hills' data is still obtained.

As mentioned earlier, it is probable that the vortex formed behind a rising bubble increases in size as it dissipates. Thus, the actual turbulence length scale may be even larger than the one given here to correct for deformation. However, the good agreement achieved (as evidenced in Figure 2) with such an elementary model seems to indicate that bubble *size* and *shape* are the controlling factors in predicting local eddy size, from which global circulation is calculated.

Table 1. Data and Predictions for Hills' (1974) Column*

Superficial Gas Velocity	Voidage	Formation Size d (mm)**	Aspect Ratio h/b Eq. 1	Minor Axis† h (mm)	Major Axis b (mm)‡
1.9 cm/s	0.07	5.0	0.807	4.34	5.37
3.8 cm/s	0.137	5.66	0.769	4.75	6.18
6.4 cm/s	0.182	6.02	0.749	4.96	6.63

* Column diameter, 14 cm; sparger, 61 holes at 0.4 mm diameter.
** Geary and Rice (1991).
† Hills (1991) used a resistance probe in computing this and presents values argued by Rice and Geary (1991) to be minor axis scales. He reported that values increase with gas velocity—2.8, 4.02 and 4.14 mm.
‡ Computed eddy size.

Notation

- b = bubble breadth, mm
- C_D = drag coefficient, dimensionless
- d = bubble formation diameter, mm
- g = acceleration due to gravity, m/s^2
- h = bubble height, mm
- ℓ = eddy length scale, mm
- Re = bubble Reynolds number, dimensionless
- Ta = Tadaki number, dimensionless
- U = bubble rise velocity, cm/s
- U_c = continuous phase interstitial velocity, cm/s

Greek letters

- ϵ = gas voidage
- $\bar{\epsilon}$ = mean gas voidage
- ξ = dimensionless radial coordinate
- ρ_L = liquid density, kg/m^3
- σ = interfacial surface tension, N/m

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